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CALIBRATION OF IMPACT TUBES

by

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Calibration of Impact Tubes

Consider a tube placed in a fluid stream, with its axis parallel to the flow, its one open end facing into the flow. Such a device causes the flow to stagnate at the tube opening, and there is developed within the tube a pressure approximately equal to the sum of the static pressure in the stream at the tube opening and the pressure developed by stagnation of the flow; viz.,

$$H = P_s + \frac{\rho u^2}{2 C_f^2} \quad (1)$$

where H is the pressure developed within the tube, P_s is the static pressure, ρ the fluid density, u the stream velocity, and C_f^2 an empirical constant. In using such a device--called an impact tube--to make measurements in jet systems, one connects the end of the tube not facing the flow to one arm of a manometer, the other arm, of which is open to the ambient pressure. The manometer reading is ΔP :

$$\Delta P = H - P_a$$

or

$$\Delta P = P_s + \frac{\rho u^2}{2 C_f^2} - P_a \quad (2)$$

where P_a is the ambient pressure. For the free jet system, the assumption is usually made that $P_s = P_a$; in such systems,

$$\Delta P = \frac{\rho u^2}{2 C_f^2} \quad (3)$$

It is usually sufficient in crude measurements to assume that C_f is unity; thus the manometer reading may be translated immediately into information dealing with the momentum flux density of the flow. If the stream density is

known, and the flow may be assumed substantially free of fluctuations, the velocity is calculable. If the measurements are to be considered precise, however, there must obviously be obtained some description of the quantity C_f .

An extensive literature survey, covering more than 400 references dating from the 18th century to the present, has disclosed few studies of the calibration characteristics of impact tubes. Goldstein (3) reported that at speeds varying from 20 to 60 ft./sec., $1/C_f^2$ did not vary from unity by more than ± 0.1 percent. At speeds from 6 to 20 ft./sec., $1/C_f^2$ varied from unity by no more than ± 1 percent. These calibrations were performed by mounting a tube at the end of a long radius, which was rotated about a center at various carefully measured speeds. The tangential velocity of the tube was thus subject to reasonably precise measurement.

It was recognized early in the program on this contract that impact tube measurements would provide a large portion of the experimental data. Calibrations were therefore attempted in order to improve the accuracy of the measurements. Values of C_f^2 appreciably less than unity were observed. The study described here was undertaken in order to explain the apparently low values of the calibration coefficients.

There appear in the literature two causes for calibration coefficients less than unity. At low velocities, Barker (2) showed that impact tubes of small radius are influenced by a viscosity effect, which may be taken into account by the equation

$$\Delta P = \frac{\rho u^2}{2} + \frac{3}{2} \frac{\mu}{r} u \quad (4)$$

where μ is the fluid viscosity and r the internal radius of the tube. Combining Equations 3 and 4, the calibration coefficient predicted at low velocities is

given by the equation

$$C_f^2 = \frac{\frac{\rho u^2}{2}}{\rho \frac{u^2}{2} + \frac{3}{2} \frac{u}{r} u} \quad (5)$$

Below are tabulated values of C_f predicted at several velocities, for $r = 0.50$ inches, in air at $70^\circ F.$ and atmospheric pressure.

u ft./sec.	C_f
4	0.971
10	0.988
20	0.994
50	0.998

For flow at high velocities, L'pmann and Puckett (5) relate the pressure drop and the kinetic head by the equation

$$\frac{2\Delta P}{\rho u^2} = 1 + \frac{1}{4} M^2 + \frac{1}{40} M^4 + \frac{1}{1600} M^6 + \dots \quad (6)$$

where M is the Mach number. Combining Equations 3 and 6 one obtains

$$\frac{1}{C_f^2} = 1 + \frac{1}{4} M^2 + \frac{1}{40} M^4 + \frac{1}{1600} M^6 + \dots \quad (7)$$

Tabulated below are values of C_f at various velocities calculated from Equation 7.

u ft./sec.	C_f
100	0.999
200	0.995
300	0.990
400	0.983

u ft./sec.	C_f
500	0.975
600	0.963
700	0.949
800	0.934

In Figure 1 are plotted the calibration coefficients listed above and those obtained experimentally by Grimmer (4) early in the program. There is obviously a serious discrepancy at low velocities. The technique of calibration must now be considered. Air was allowed to flow through a carefully built nozzle with an A.S.M.E. elliptical approach contraction. The temperature and pressure upstream of the nozzle were measured. By means of a calibration of the nozzle, previously shown to be correct to 0.1 percent, the mass flow through the nozzle was determined. An impact tube was located on the centerline of the nozzle, its open end about 1/4 inch from the nozzle exit. The assumption was made that the velocity was everywhere the same in the plane of the nozzle discharge. On the basis of the known area of the nozzle discharge, the assumed uniform velocity profile, and the measured flow rate, the velocity at the centerline of the nozzle could be computed. The impact head, as measured by the impact tube, was recorded. The impact head and the kinetic head, so determined, were compared, and values of C_f calculated, according to Equation 3.

The obvious flaw in this procedure lies in the assumption of a uniform velocity profile in the plane of the nozzle discharge. Two questions must be raised: Is it reasonable that the velocity profile be uniform? If not, how should that profile vary as a function of the velocity at the centerline?

Consider a momentum balance between two sections through the discharge nozzle. These sections are perpendicular to the centerline of the nozzle, with Section 1 upstream from the nozzle contraction, and Section 2 coincident with the nozzle discharge. Let u_1 be the velocity at the centerline at 1, u_2 the velocity at the centerline at 2, V_1 the mean velocity across Section 1, and V_2 the mean velocity across Section 2. By Bernoulli's theorem,

$$\frac{u_2^2 - u_1^2}{2} = \Delta P$$

Similarly,

$$\frac{V_2^2}{2} - \frac{V_1^2}{2B^2} = \Delta P$$

where B^2 is the skewness coefficient required to extend Bernoulli's theorem to the mean velocities. At 1, V_1 and u_1 are related by the equation $V_1 = \alpha u_1$, where α is a constant characteristic of the flow in the nozzle at Section 1.

If the variation in density between 1 and 2 is small, then by a material balance, $V_1 S_1 = V_2 S_2$, where S_1 and S_2 are the areas of the two sections of the nozzle at 1 and 2, respectively. Then

$$u_2^2 - \frac{V_1^2}{\alpha^2} = V_2^2 - \frac{V_1^2}{B^2} = V_1^2 \left[\left(\frac{S_1}{S_2} \right)^2 - \frac{1}{B^2} \right]$$

and

$$u_2^2 = V_1^2 \left[\left(\frac{S_1}{S_2} \right)^2 + \frac{1}{\alpha^2} - \frac{1}{B^2} \right]$$

Replacing V_1^2 with its equivalent in terms of V_2 ,

$$u_z^2 = v_z^2 \left(\frac{S_2}{S_1} \right)^2 \left[\left(\frac{S_1}{S_2} \right)^2 + \frac{1}{\alpha^2} - \frac{1}{B^2} \right]$$

$$= v_z^2 \left[1 + \left(\frac{S_2}{S_1} \right)^2 \left(\frac{1}{\alpha^2} - \frac{1}{B^2} \right) \right] \quad (7)$$

Only when the quantity in the bracket is unity is the mean velocity across the nozzle discharge exactly equal to the velocity at the centerline.

For the flow nozzle used in the impact tube calibrations, the upstream diameter was 1.610 inches and the discharge diameter was 0.952 inches. The ratio S_2/S_1 is 0.35. The value of B is usually close to 1; and is a function of the flow rate through the nozzle. If the flow at 1 were normal turbulent flow in a pipe, the value of α would be about 0.8. In that case, the quantity in the bracket would have the value $1 + [(0.35)^2 (1.56 - 1)]$ or 1.068. As the velocity increases, the velocity profile upstream of the nozzle should flatten, and α should approach unity. The mean velocity at the discharge then should be equal to the centerline velocity. Conversely, at low velocities, α is less than 0.8, and the mean velocity at the discharge should differ still more from the centerline velocity. Thus, it should not be expected that the velocity profile which existed under the conditions of calibration at low velocities should have been flat, as assumed in the calculation of C_F .

In order to obtain an estimate of variation in velocity profile as a function of centerline velocities the following assumptions are made:

When the centerline velocity is u_0 in a nozzle of radius r_2 , the velocity profile is flat at the value u across a circular area of radius r_1 , located in the center of the nozzle. In the annular region for which the inner radius is r_1 , and outer radius r_2 , the velocity varies from u_0

when $r = r_1$ to 0 when $r = r_2$. The variation is parabolic, such that

$$\frac{du}{dr} = 0 \text{ when } r = r_1.$$

The above assumptions can be used to obtain analytic expressions for the velocity as a function of r . Since the velocity variation has been assumed to be parabolic,

$$u = a + br + cr^2. \quad (8)$$

By employing the assumptions above, the constants of Equation 8 can be evaluated to give

$$u = u_0 \left(1 - \left[\frac{r - r_1}{r_2 - r_1} \right]^2 \right) \quad (9)$$

Let the volumetric flow through the nozzle be q ; then $q = \int_0^{r_1} 2\pi r u dr$

$$q = 2\pi u_0 \int_0^{r_1} r dr + 2\pi u_0 \int_{r_1}^{r_2} \left[1 - \left(\frac{r - r_1}{r_2 - r_1} \right)^2 \right] r dr \quad (10)$$

Evaluation of Equation 10 gives

$$q = \pi u_0 \left[r_2^2 - \frac{1}{6} (3r_2 + r_1)(r_2 - r_1) \right] \quad (11)$$

Now let $r_2 - r_1 = \delta$, where δ is the film thickness. Then

$$3r_2 + r_1 = 4r_2 - \delta \quad (12)$$

Substituting in Equation 11

$$q = \pi u_0 \left[r_2^2 - \frac{1}{6} (4r_2 - \delta) \delta \right] = \pi u_0 \left(r_2^2 - \frac{2}{3} r_2 \delta \right) \quad (13)$$

ignoring the higher power of δ .

When the impact tubes were calibrated at first, on the assumption of a completely uniform velocity profile, the centerline velocity u_c' was

obtained from the equation

$$q = \pi r_2^2 u_0' \quad (14)$$

Following the derivation above, assuming a finite boundary layer of thickness δ , the centerline velocity u_0 was described in Equation 13. Eliminating q between 13 and 14 and rearranging,

$$\frac{u_0'}{u_0} = 1 - \frac{2}{3} \frac{\delta}{r_2} \quad (15)$$

The impact tube coefficients are proportional to the measured velocities according to Equation 3, so

$$\frac{C_f'}{C_f} = 1 - \frac{2}{3} \frac{\delta}{r_2} \quad (16)$$

where C_f' and C_f are the impact tube coefficients obtained from the assumption of flat and variable velocity profiles, respectively. Using the impact tube coefficients measured by Grimmer (4) for the C_f' and the values calculated from Equations 5 and 7 for the C_f , it is possible to compute the value of $\frac{\delta}{r_2}$ at several velocities. These are tabulated below, as well as the values of δ corresponding to Grimmer's measurements.

u_0	δ/r_2	(Grimmett)
20 ft./sec.	.131	.086 in.
50	.151	.072
100	.120	.057
200	.091	.043
400	.047	.023
600	.023	.011

Experimental

The ultimate purpose of experimentation in this field was the construction of an apparatus suitable for the calibration of impact

tubes, and thus to corroborate the equations already obtained and experimentally verified by Barker, (2) and by Liepmann and Puckett (5).

A suitable apparatus could have the following characteristics:

(1) An orifice flow meter with a precision of ± 0.5 percent at all points in the velocity range.

(2) A calibrating nozzle with a contraction ratio such that $\left(\frac{S_2}{S_1}\right)^2$ as in Equation 7 would be less than 0.001, and of such a diameter that the film region would be appreciably thicker than the impact tube used to traverse the nozzle.

(3) A traversing mechanism capable of positioning an impact tube to within 0.001 inch, immediately downstream of the nozzle discharge, without blocking the flow.

A prototype for such a calibration system was built. The orifice meter consisted of nine brass plates, each with a stainless steel disc at its center, in which was drilled an orifice following the recommendations of the A.S.M.E. (1). Photomicrographs were made of the plates, and the orifice areas were measured by a planimeter with the assistance of a scale which was part of each photomicrograph. The orifice plates were then calibrated according to a flow nozzle previously found to be correct to ± 0.1 percent. The precision of the orifice meter was within the limits set.

It was decided that the calibrating nozzle should have an upstream diameter twelve times the discharge diameter; thus $\left(\frac{S_2}{S_1}\right)^2 = \left(\frac{1}{144}\right)^2$,

well within the stated limit. The largest pipe available was standard six-inch pipe; hence the nozzle diameter was fixed at 0.500 inch. The smallest tubing available for the impact tube was 1/32-inch o.d. stainless steel hypodermic tubing. The traversing device consisted essentially of

two screws arranged perpendicular to each other, permitting adjustment in planes normal to the nozzle geometric axis. Each screw had a pitch of 0.05 inches and a hand wheel whose circumference was marked off in fifty equal divisions.

The experimental procedure started with locating the center of the nozzle with respect to the traversing device. A plug with a $1/64$ -inch hole at its center was held by spring action in the nozzle; thus the hole was believed to be at the center of the nozzle. Air was admitted upstream of the nozzle and escaped through the hole. The impact tube, mounted in the traversing device, was moved until the point was located at which the pressure was at a maximum. Repeated trials showed that the reproducibility of the centering procedure was ± 0.005 inches.

The nozzle center being known, the centering plug was removed from the nozzle. The upstream pressure was adjusted to provide the desired volumetric flow through the nozzle. At this point, the object was to determine the extent of the region of constant impact pressure immediately downstream from the plane of discharge of the nozzle. Obviously no measure of velocity could be made, either in the region of constant pressure or in the film region where the pressure fell off, for the impact tube used in the measurements had not been calibrated. Thus no exact calculation could be made of the velocity distribution across the nozzle, nor was any intended.

The impact tube traversed the nozzle in two directions, and a pressure profile was obtained. This procedure was repeated at each of several flow rates. In each case, a large region of constant pressure was noted about the nozzle center, and close to the edge the pressure was observed to fall off. The "film" region was presumed to begin at the point at which the pressure was 3 percent below the average value

measured in the constant pressure zone.

The observed "film thickness" at each velocity and the value of δ/r_2 corresponding to it are tabulated below. The observed δ/r_2 is compared with the value of δ/r_2 calculated from Equation 16. The experimental data of Grimmett were used in Equation 16.

u	δ/r_2 (Eq. 16)	δ/r_2 (Measured)	δ (Measured)
20 ft./sec.	0.181	0.24	0.06 inch
50	0.151	0.10	0.025
100	0.120	0.088	0.022
200	0.091	0.070	0.0175
400	0.047	0.060	0.0150
600	0.023	0.060	0.0150

The agreement between the recently observed data and those obtained from Grimmett's measurements by Equation 16 is not sufficient to confirm beyond question the hypotheses presented earlier. The recent measurements do indicate the presence of a slow-moving boundary layer, of which the thickness diminishes as the gas flow through the nozzle goes up. This confirms, in general, the explanation previously offered for the unusually low impact tube calibration coefficients observed by Grimmett. However, the discrepancies between the computed δ/r_2 and the measured values do preclude the use of the current apparatus for calibration of impact tubes. The discrepancies may be explained in part as follows.

First, it should be noted that at velocities higher than 200 ft./sec., δ changes only slightly. The value observed is .015 inches; the impact tube used had an outside diameter of 1/32 inch or about .0313 inches; the tube opening was about half that, or .015 inches. Thus the magnitude of the film at velocities beyond 200 ft./sec. is of the order of the diameter of the tube opening, and therefore measure-

ments of a smaller film thickness are not possible. Moreover, in view of the total film thickness even at the low velocities, the relative error is an appreciable part of the total thickness.

It appears that Equation 7 does not apply quantitatively. Even though $(S_2/S_1)^2$ is small, the centerline velocity u_0 is appreciably greater than the mean velocity calculated from the integrated velocity distribution given by Equation 14. The ratio $\frac{V}{u_0}$ as a function of u_0 is tabulated below.

u_0 (ft./sec.)	V/u_0
20	0.84
50	0.93
100	0.94
200	0.98
400	0.98
600	0.98

Conclusions

It may be concluded that the hypothesis regarding the existence of a relatively slow-moving film close to the wall of the flow nozzle is reasonably well substantiated. This explains the unusual impact tube coefficients which Grinnett observed.

As to the use of the present apparatus for the calibration of other impact tubes, it must be concluded that this is not possible. The nozzle diameter is too small, relative to the diameter of the tube used to determine the pressure profile. Since that tube is already as small as practicable, the size of the nozzle must be increased. Equation 7 is apparently a gross oversimplification, since a far larger boundary layer was observed than the equation would indicate. It is therefore probable that the 12/1 contraction ratio is not necessary.

There is at hand, therefore, no device which permits calibration

of impact tubes with the requisite precision. Two alternatives are apparent: (1) to assume that $C_f = 1.0$; or (2) to accept the validity of the coefficients predicted by Barker, and by Liepmann and Puckett; i.e., the coefficients calculated from Equations 5 and 7. The latter course was adopted in most of the work in this program.

Nomenclature

- a An arbitrary constant in Equation 3.
- b An arbitrary constant in Equation 6.
- B^2 Skewness coefficient in Equation 7.
- C An arbitrary constant in Equation 8.
- C_f Impact tube calibration coefficient, Equation 1.
- C_f' Coefficient based on the assumption of uniform velocity across flow nozzle.
- H Pressure in impact tube.
- M Mach number, dimensionless.
- P_a Ambient pressure.
- P_s Static pressure.
- ΔP Pressure difference indicated by impact tube .
- q Volumetric flow through nozzle.
- r Radial coordinate.
- r_1 Inner radius of slow-moving film.
- r_2 Radius of flow nozzle.
- S_1 Upstream area of flow nozzle.
- S_2 Downstream (contracted) area of flow nozzle.
- u Velocity at a point in the discharge of a flow nozzle.
- u_0 Velocity at the centerline.
- u_0' Spurious centerline velocity, $q/\pi r_2^2$.

- u_1 Velocity at $r = r_1$.
- u_2 Velocity at $r = r_2$.
- V Mean velocity in a transverse section.
- α Relationship between centerline and mean velocities.
- δ Thickness of the slow-moving film, $r_2 - r_1$.
- μ Viscosity of fluid.
- ρ Density of fluid.

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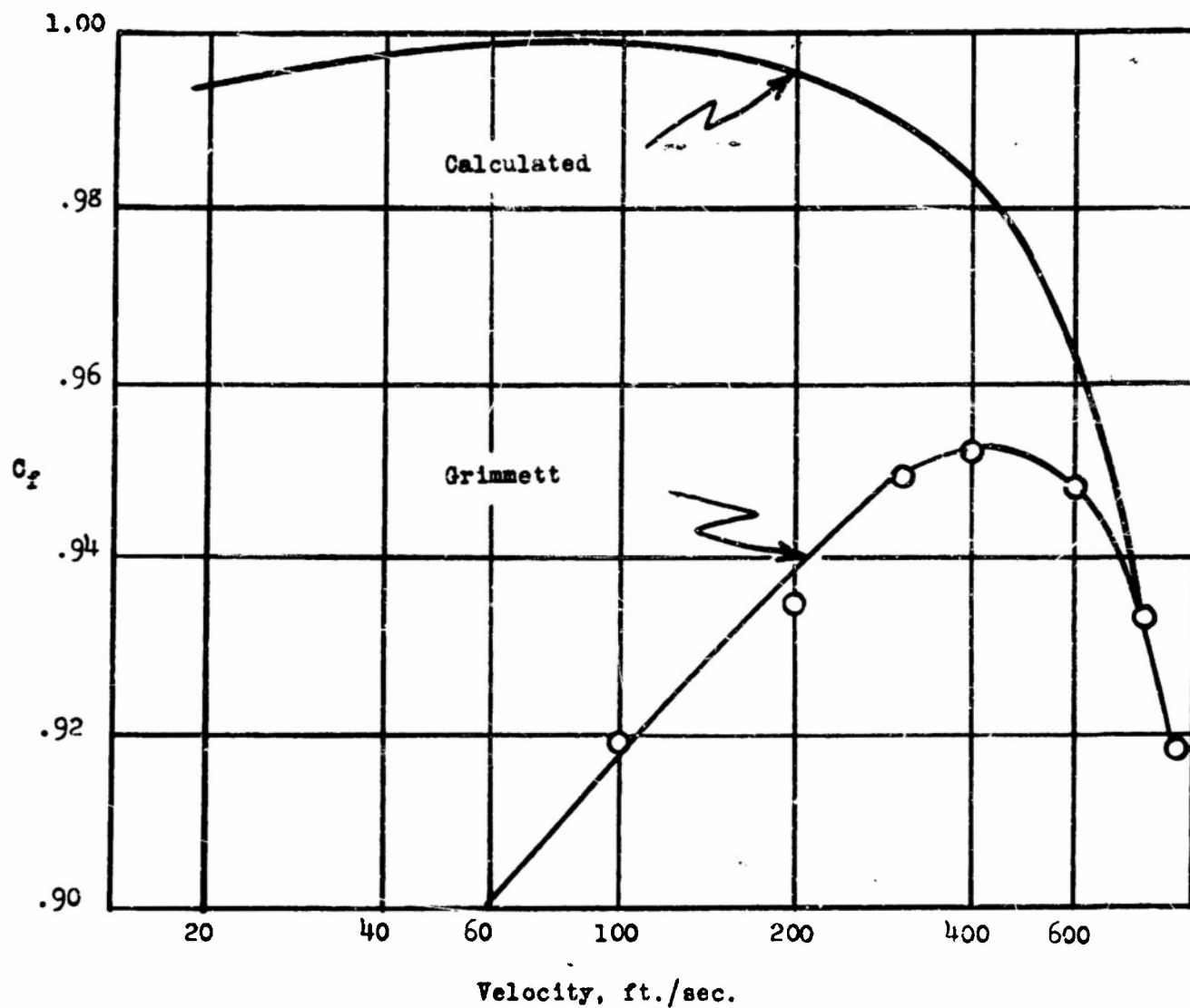


Figure 1
Calibration Coefficients for Impact Tubes